Optimal Retirement Income Tontines:

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York University, Toronto

18 September 2015 (MIT)
1. Practical Motivation
   - Mortality-Contingent Claims In the News.

2. Some Early History
   - What exactly is a tontine?
   - Who invented this weird scheme?
   - How does it differ from a life annuity?

3. Optimal Design of Retirement Income Tontines
   - Should they be resurrected (in some modified form)?
   - What Would a Modern Day Tontine Look Like?

4. Conclusion and Final Thoughts
Outline

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4 Conclusion and Final Thoughts
Uncle Sam Wants You....To Buy a Life Annuity: But does anybody else?

FOR IMMEDIATE RELEASE: October 24, 2014
CONTACT: Erin Donar, Treasury Public Affairs (202) 622-2690

TREASURY ISSUES GUIDANCE TO ENCOURAGE ANNUITIES IN 401(k) PLANS
New option for plan sponsors to include income annuities in default target date fund investment options

WASHINGTON – In order to help retirees manage their savings and ensure they have a stream of regular income throughout retirement, the U.S. Department of the Treasury and the Internal
Remaining Lifetime (RL) Random Variable (RV)

- Healthy Female: 10 years
- Unhealthy Male: 8 years

Longevity Risk
- Healthy Female: 10 years
- Unhealthy Male: 8 years

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Mid 17th Century: England & France
An era before government bonds

Example: Renaissance Tontine with 10% Annual Dividend

- **Annuitant**, Subscriber, Investor (Rich Male)
- **Nominee** (Young Female)
- Dividend to Annuitant While Nominee is Alive
- Payment Obligations End
- Nominee Dies
- Tontine Issuer, Sponsor or Government

- £100
- £10+?
- £10+?
- £10+?
- £0

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Optimal Retirement Income Tontines: 18 September 2015 (MIT)
Example: Classic Tontine with 20 Investors

(= Nominee = Annuitant)
Example: Classic Tontine with 20 Investors
After Some Deaths

Example: Classic Tontine with 20 Investors
With 16 investors alive and 4 investors dead, the dividend calculation is:

\[ \frac{\£200}{16} = \£10 + \£2.5 = \£12.5 \]

At the end of the year, the dividend per share is (i.) interest of 10% plus (ii.) mortality credits of 2.5%. It is crystal clear what the tontine pays.
Dividend per Share

Why the revisions?

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Dividend per Share: 4 Investors Remaining in the Pool

£200 ÷ 4 survivors = £50 dividend per survivor
Dividend per Share: The Last One Standing

Winner of the Tontine in Year #?

Winner!

£ 200 Annuity for life...
What Happens to the Principal?

...It is gone!
What Happens to the Principal?

...It is gone!
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A very clever Italian...

**Promoter: Lorenzo de Tonti (1602 – 1684)**

- Banker & Governor of Gaeta, Italy.
- Involved in revolt against Spanish and sought asylum in France in 1650s
- Introduced tontines in 1653 when he wrote *‘Edict of the King for the creation of the Society of the Royal Tontine’*
- Imprisoned from 1668 – 1675 in the Bastille for reasons unknown.
- He likely died in poverty and never participated in a tontine!
The Only Known Picture

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Compare and Contrast

Tontine vs. Life Annuity

Tontine

- Total interest paid each year to group known in advance
- Payments to survivors increase...and the longer they live the more they get!

Life Annuity

- As people die, total paid out to group declines over time
- Payments and income to survivors stays relatively constant
Adam Smith: “Tontine Appeals to the Gambling Instinct”

"...from the confidence which every man naturally has in his own good fortune, the principle upon which is founded the success of all lotteries, [the tontine] annuity generally sells for something more than it is worth. In countries where it is usual for government to raise money by granting annuities, tontines are upon this account generally preferred to annuities for separate lives..."
Imagine you (are about to retire and) are given a choice between a 14% life annuity and a tontine paying 8% guaranteed. Which would you select to hedge your personal longevity risk?
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What would (rational) choice depend on?

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Tontine or Life Annuity
Which Would You Pick?

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✓ Your health relative to the tontine pool.
✓ Credit rating of the insurance company.
✓ Risk aversion & consumption preferences.
✓ Pricing and the term structure of interest rates.
This Isn’t Hypothetical...
In the year 1693 investors in England were given this choice

King William III & Queen Mary II

England

King Louis XIV

France

The Million Act...to finance the war with France

Choice A
€100

Annuity
14% For Life

Choice B
€100

Tontine
10% for 7 years
7% thereafter

16%
12%
8%
4%

1693 1700 1705 1710 1715
Annual Reports & Dividend Statements
Documents Stored in the National Archives and British Library
Actual Choices Between a Tontine and a Life Annuity

In the year 1693, did investors have access to financial advice?

<table>
<thead>
<tr>
<th>Investor Characteristics</th>
<th>Invested in Annuity</th>
<th>Invested in Tontine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Investment</td>
<td>£269,500</td>
<td>£108,100</td>
</tr>
<tr>
<td>Number of Annuitants</td>
<td>1,015 (60%)</td>
<td>665 (40%)</td>
</tr>
<tr>
<td>Number of Investors</td>
<td>2,525</td>
<td>1,013</td>
</tr>
<tr>
<td>Investors from London</td>
<td>71%</td>
<td>29%</td>
</tr>
<tr>
<td>Investors outside London</td>
<td>35%</td>
<td>65%</td>
</tr>
<tr>
<td>Average Investment Size</td>
<td>£265.46</td>
<td>£162.56</td>
</tr>
<tr>
<td>Largest Subscriber</td>
<td>Sir Robert Howard</td>
<td>William Tempest</td>
</tr>
<tr>
<td>Largest Subscription</td>
<td>£4,200</td>
<td>£1,600</td>
</tr>
<tr>
<td>Investors with ≥ 10 Shares</td>
<td>95%</td>
<td>5%</td>
</tr>
</tbody>
</table>

The Astronomer Edmond Halley thought 14% was a better deal...

This shows the great advantage of putting money into the present fund granted to their majesties, giving 14% per annum, at the rate of 7 years purchase for a life, when [even] the young lives at the annual [6%] rate of interest, are worth above 13 years purchase...

On the off chance you are interested...

The book was published by Cambridge University Press in May 2015.

In a time before bonds, treasuries, notes, or central banks, there were annuities. These were schemes in which a group of investors' money was a government, corporation, or similar to a modern day bank. But unlike conventional debt, periodic interest payments were distributed only to survivors. As some investors died, the income of survivors correspondingly increased. Morbid, perhaps, but this was one of the earliest forms of longevity insurance in which the problem of risk was managed.

Moshe Milevsky tells the story of the first tontine issued by the English government in 1699. Known as King William's tontine, it intended to finance the war against France's King Louis XIV. He explains how contingent work, the financial and economic thinking behind them, as well as why they fell into disfavor. Milevsky concludes with a provocative argument that manly modified annuities should be structured for every-day consumer income planning.

MOSHE A. MILEVSKY is a Professor at the Schulich School of Business at York University and Executive Director of the nonprofit ERM Centre at the Fields Institute for Research in Mathematics in Toronto. He is also Managing Director of the P.E. I. Longevity Exchange (PEICLE) Corporation, which is focused on improving pension funds, endowments, and individuals extend the longevity of their investment portfolios. In 2005, he founded the insurance company, QNMA Group, which he recently sold to CANVEX Financial Exchange. He has published over ninety peer-reviewed articles, hundreds of newspaper and magazine columns, and currently on the editorial board of numerous academic journals. This is his twelfth book.

King William's Tontine

Why the Retirement Annuity of the Future Should Resemble Its Past

MOSHE MILEVSKY

Cambridge University Press
Why Don’t They Exist Anymore?

- Why don’t they exist anymore?
  - Centenarians with (winner-takes-all) millions makes no sense.
  - Concerns with fraud and possible criminal incentives.
  - Historical (18th century) complaints about investment returns.
  - Early 20th century ban (in U.S.) on similar type of insurance.
  - Distasteful and unethical, etc.

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✓ Distasteful and un-ethical, etc.
The (Expected) Payout Structure of a Flat Tontine
Not Very Optimal Looking...

Range of Flat 4% Tontine Payout Purchased at 65: Gompertz Mortality
10th vs. 90th percentile: n = 400 (m=88.721, b=10)
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Who Should Bear Systematic Longevity Risk?
And how much will it cost?

U.S. Mortality Rates at Age 85: During Period 1975 to 2010
Source: Human Mortality Database
Why Bother Resurrecting Them?
The Cost of a Longevity Guarantee in a Life Annuity is Not Cheap
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What is An Optimal Tontine?
Some Quick Intuition

Many ways to design a tontine in which the PV of the guaranteed dividends is equal to the original contribution.

IV = Optimal Tontine....
A Formal Derivation of the Optimal Tontine

The main contribution of the research paper...

The optimal tontine is

$$D(t_p x) = D(1)^{\beta(t_p x)}/\gamma$$

where

$$D(1) = \left[ \int_0^\infty e^{-rt\beta(t_p x)^{1/\gamma}} dt \right]^{1/\gamma}.$$ 

More generally we have that for any

$$\beta_n, \gamma(p)$$ is

$$\begin{cases} < p^{\gamma}, & \gamma < 1 \\ = p^{\gamma}, & \gamma = 1 \\ > p^{\gamma}, & 1 < \gamma \end{cases}$$

The optimal payout rate is identical to the survival probability (only) when utility is logarithmic. We think it is a natural way to build a modern day retirement income tontine.
The main contribution of the research paper...

✓ The optimal tontine is $D(t p_x) = D(1) \beta(t p_x)^{1/\gamma}$, where

\[
D(1) = \left[ \int_0^\infty e^{-rt} \beta(t p_x)^{1/\gamma} dt \right]^{-1}.
\]
The optimal tontine is $D(tp_x) = D(1)\beta(tp_x)^{1/\gamma}$, where

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More generally we have that for any $n$ and $0 < p < 1$,

$$\beta_{n,\gamma}(p) \begin{cases} < p^\gamma, & 0 < \gamma < 1 \\ = p^\gamma, & \gamma = 1 \\ > p^\gamma, & 1 < \gamma \end{cases}$$
A Formal Derivation of the Optimal Tontine

The main contribution of the research paper...

✓ The optimal tontine is $D(tp_x) = D(1)\beta(tp_x)^{1/\gamma}$, where

$$D(1) = \left[ \int_0^\infty e^{-rt} \beta(tp_x)^{1/\gamma} \, dt \right]^{-1}.$$  

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✓ The optimal payout rate is identical to the survival probability (only) when utility is logarithmic. We think it is a natural way to build a modern day retirement income tontine.
The Natural Tontine
More Now, Less Later and Expectations are Flat

Natural Tontine Payout Function: Cohort of Size 250 at Age 65
Risk Free Rate = 4%, Gompertz (m = 88.721, b = 10)
The Range of Outcomes in an Optimal Tontine

Nothing is certain in life, but...

Range of **Optimal** Tontine Payout at **4% Interest**: Gompertz Mortality
10th vs. 90th percentile: $n = 400$ ($m=88.721$, $b=10$)
If annuity loading is too high...I might prefer a tontine

Think Annuity Equivalent Wealth (AEW)

<table>
<thead>
<tr>
<th>Risk Aversion</th>
<th>$n = 20$</th>
<th>$n = 100$</th>
<th>$n = 500$</th>
<th>$n = 1000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 0.5$</td>
<td>72.6 b.p.</td>
<td>14.5 b.p.</td>
<td>2.97 b.p.</td>
<td>1.50 b.p.</td>
</tr>
<tr>
<td>$\gamma = 1.0$</td>
<td>129.8 b.p.</td>
<td>27.4 b.p.</td>
<td>5.74 b.p.</td>
<td>2.92 b.p.</td>
</tr>
<tr>
<td>$\gamma = 1.5$</td>
<td>182.4 b.p.</td>
<td>39.8 b.p.</td>
<td>8.45 b.p.</td>
<td>4.31 b.p.</td>
</tr>
<tr>
<td>$\gamma = 2.0$</td>
<td>231.7 b.p.</td>
<td>51.8 b.p.</td>
<td>11.1 b.p.</td>
<td>5.68 b.p.</td>
</tr>
<tr>
<td>$\gamma = 3.0$</td>
<td>323.1 b.p.</td>
<td>75.1 b.p.</td>
<td>16.3 b.p.</td>
<td>8.38 b.p.</td>
</tr>
<tr>
<td>$\gamma = 9.0$</td>
<td>753.6 b.p.</td>
<td>199.8 b.p.</td>
<td>45.9 b.p.</td>
<td>23.8 b.p.</td>
</tr>
</tbody>
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Assumes Age $x = 60$, $r = 3\%$ and Gompertz Mortality ($m = 87.25$, $b = 9.5$)

✓ **Problem:** How do you (plan a) smooth retirement if you don’t know how long you are going to live?
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✓ Insurance companies **charge** for absorbing longevity risk. This (capital) cost will continue to increase over time.
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✓ Perhaps offer (portfolio) **choice** between (i.) sharing longevity risk with others, and (ii.) transferring to an insurance company.
✓ **Problem:** How do you (plan a) **smooth** retirement if you don’t know how long you are going to live?

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✓ Perhaps offer (portfolio) **choice** between (i.) sharing longevity risk with others, and (ii.) transferring to an insurance company.

✓ We bring some (i.) economic rationale and (ii.) transparency to the design of participating annuities by introducing the **natural retirement income tontine.**